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Westinghouse

ELECTRIC CORPORATION



AIR ARM DIVISION

PHONE: SOUTHFIELD 1-1000
FRIENDSHIP INT'L AIRPORT
BOX 746, BALTIMORE 3, MD.

January 25, 1962

A18056S

Special Projects Office
(IMBA-1)
Plans & Programs Office
Directorate of Production
Wright-Patterson AFB, Ohio

SUBJECT: Monthly Progress Report
Contract AF 33(600)-40260

Enclosure (1): Three (3) copies Monthly Progress Report for Period from
November 15, 1961 through December 15, 1961.

Gentlemen:

Enclosure (1) is submitted as required by the subject contract.
One copy of this report is also being sent to [redacted]

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Very truly yours,

WESTINGHOUSE ELECTRIC CORPORATION

[redacted]
Director of Programming
APQ-93 Program

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cc: [redacted]

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IF ENCLOSURES ARE WITHDRAWN (OR NOT ATTACHED), THE
CLASSIFICATION OF THIS CORRESPONDENCE WILL BE CAN-
CELED IN ACCORDANCE WITH PAR 251 AF REGULATION 25-4
OR NAVY REGULATION ARTICLE 25 (5) (D).

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Encl #1
OXC-3003
COPY 3 OF 3

Progress Report

Period of 11-15-61 to 12-15-61

Contract No. AF33(600)40280

General

Major activities for the monthly reporting period:

1. Composite testing of radar set #1.
2. Modification and cabling of F101 aircraft.
3. Preparation of #2 radar set for composite test.

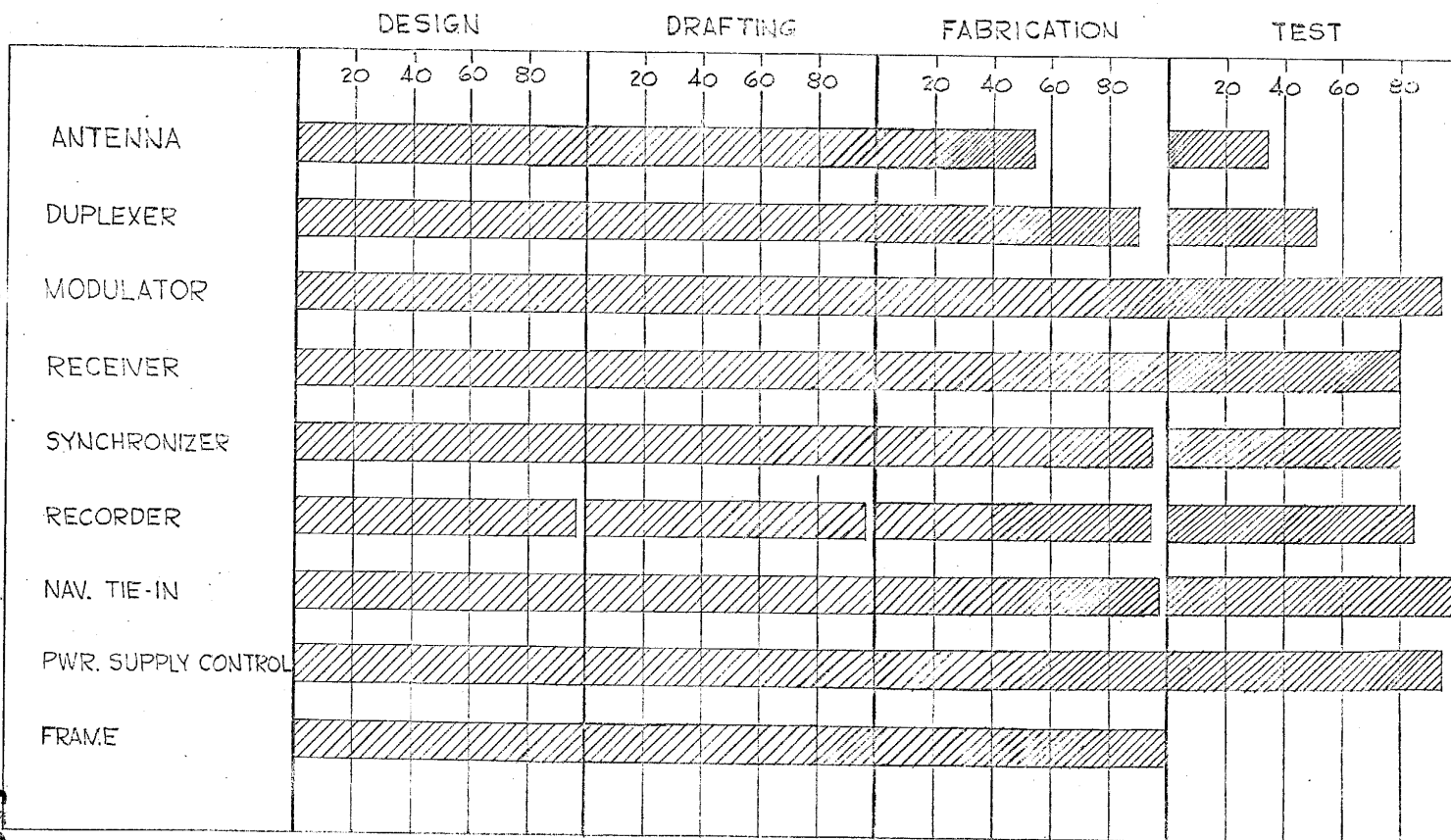
The chart (page 2) shows in graphical form the status of the first radar system.

Composite test of the #1 radar set is proceeding in time with the schedule calling for delivery by January 1. Acceptance sign-off is planned for December 28 with the exception of the antenna. The antenna as modified will be available for installation in the aircraft by the beginning of the flight test program.

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PROJECT STATUS

% COMPLETION OF 1ST RADAR FOR PERIOD ENDING 15 DEC. 1961



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Flight Test

General

With the assistance of a Minneapolis-Honeywell representative, and auto-pilot system in the F101-B was checked out. A defective roll rate gyro was found to be the cause of the roll instability experienced during the pilot check-out flights. Installation of a new stabilizer servo eliminated the aircraft altitude drift which had previously been encountered when operating in the "altitude hold" mode.

Improved performance of the auto-pilot after the incorporation of these changes was apparent during the two flights made to test aircraft stability. These flights were made to test operation of the system and to determine the mode of auto-pilot operation which best suits the stability requirements of this program. "Altitude Hold" appears to be the best mode of operation.

Immediately after the two auto-pilot flights (November 22 and November 24) aircraft modification and equipment installation began. This phase of the program is continuing at the end of the reporting period.

Auxiliary and Ground Support Equipment

Out of a total of 87 items of G.S.E. requested to date, all but four have been received. The remaining four items are not critically needed and are understood to be in transit. Table 2 and Table 3 (aircraft and engines) spares lists have been scrutinized, along with individual parts breakdown (IPB) for selection of a group of spares for normal stock. These spares have been divided into priority categories and forwarded to AFIC for action.

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Instrumentation

Fabrication of instrumentation units is 70% complete.

Schedule

The schedule has been modified to move the fly date to the originally planned date of February 2, 1962. Key dates are:

January 1	APQ-93 delivery to Test and Evaluation
January 9	Complete system installation excluding pod, begin system ground test.
January 26	Complete systems test.
February 2	Flight

System

Lens Optics Recorder

An Eastman lens similar to the chosen lens was received on loan and tested. A resolution in excess of 140 lines/mm was measured on the aerial image. No measurement of flatness of field was possible due to equipment difficulties. A resolution in excess of 70 lines/mm on 5374 film was achieved, and it was felt that by varying exposure a higher resolution could have been obtained. The tests were cut short by the necessity of returning the lens to Eastman.

A review of the feasibility of the lens optic system was conducted. It was decided that this approach fell short for the design conditions, but was promising as a back up for the flight test program and effort should be continued to that end.

A comprehensive analysis of the response of the cathode ray tube to sinusoidal and pulse type signals has been prepared and is appended.

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Antenna

Array Design - Radome Laminate Design

The ML fabric - EC1663 adhesive sealing technique is being employed satisfactorily for sealing array sticks for the flight test antenna.

An additional investigation of high temperature sealing techniques applicable to slotted array sticks is being conducted by the materials and processes section. Three areas are being investigated: (1) high temperature adhesive bonding, (2) producing a dielectric film cover by multiple applications of a varnish solution, (3) electron beam welding a nickel - dielectric "sandwich" onto waveguide.

Fabrication - Flight Test Antenna

Beam: Of the three stainless steel honeycomb beams received, two were returned to the supplier for alteration to improve flatness. Beam was flat to within 0.125 in. instead of 0.060 in. as specified. The third beam was retained pending alteration of the others. Beam number 1 was altered, and has been inspected and accepted. Beam number 2 was altered by "heat creeping" and is presently being inspected.

Manifolds: A total of four manifolds for the flight test antenna has been received to date. These have satisfactorily passed electrical and dimensional checks. X-ray inspection by the supplier prior to shipment has been beneficial in insuring complete removal of the mandrels.

Two remaining manifolds for the flight test antenna are being gold plated by the supplier and have been promised on December 21.

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Array Sticks: A total of 88 useable sticks has been received from the supplier to date; 83 of these are useable in flight test antenna. The remaining 13 sticks for the flight test antenna have been promised by December 21.

All flight test array sticks presently on hand have been sealed.

Modules: Three modules of the six required for the flight test antenna have been assembled and tested preparatory to being "grown together". Two of these modules were transported to the supplier on December 13 to be grown together; the third will be transported on December 21. A fourth module is ready for assembly with the exception of one stick.

The process of electroforming an assembled module into an integral part appears to be progressing satisfactorily. Despite experimental work done at Westinghouse-Pittsburgh with Heliarc brazing of modules, the electroforming process appears to be the only joining process available which will provide a reasonable bond on the narrow faces of the sticks as well as on the broad faces.

Electrical tests of assembled modules have been satisfactory.

Power Dividers: Sufficient electroformed parts were promised for delivery to subcontractor on or before December 20. Approximately one week will be required for the subcontractor to assemble sufficient parts for the flight test antenna.

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Duplexer

All three duplexer units have been assembled and modifications due to many possible final switch configurations are now in progress on units 2 and 3. One of these units will be mounted on Frame #1 upon completion while unit 1 is modified.

One EGG switch tube has been received and exhibited the following characteristics during operation:

1. Sweeping electrodes (+2000V at ≈ 1 va) showed it to be adequate for clean up.
2. Hold-off on one side was 50 KW, trigger controlled. Above this value, windows arc over internally. Hold-off on the other side was only 10 KW due to non-concentricity of domes and sweeping electrode hole.

It is expected that EGG will correct these difficulties and supply another tube during the week of December 24, 1961.

A cover, to prevent RI pick up in the receiver due to switch pulsing is being constructed.

Duplexer Driver

Unit #1 has been mounted in the frame and modifications to provide +2000V sweeping voltage will be made.

Unit #2, testing has been completed.

Unit #3 is complete except for testing.

Power Monitor

One unit is complete and has been mounted in Frame #1. The two remaining units are now being tested.

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Switch Tubes

A dual tube was built for use at high pressure which had one isolated dome structure. It was anticipated that with this method for triggering, an arc could be established readily across the gap and more effectively control the firing and reduce arc loss. During the tests the insulation was damaged due, it is believed, to a corona discharge that existed around the feed through hole in the waveguide. As a result, evaluation could not be completed. This was to have been the second tube delivered to Engineering.

At present, another tube is being constructed in which ultraviolet triggering will be used. Instead of a dome type gap previously used, a planar type of resonant structure will be used in an attempt to see if the insertion loss of the tube can be reduced. This tube will have an input and output window.

A third tube is being built which will be used to investigate more completely some observations made previously under low pressure conditions. At the time, the tube used had a very short life so that sufficient data could not be taken. The present tube is being carefully prepared and processed in an effort to improve life so that the low pressure switch can be evaluated.

Modulator

A pulse transformer PFN was received which is suitable for operation of the first modulator, however, redesign of the PFN is now underway by the supplier to provide improvements in pulse width and impedance matching to the klystron. When available, the redesigned unit can be incorporated in the first modulator.

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Receiver

TWT

The first TWT assembly is on Frame #1 and is now in composite test. The second assembly is being placed on Frame #2.

A filter box is being designed to mount near the TWT assembly in order to eliminate some pickup which was discovered on the TWT power supply leads.

I.F. Amplifier

The first unit is now in composite test, the second and third units have been unit tested and the second is being mounted on Frame #2.

Video Amplifier

The first unit is now in composite test, the second has been through unit test and the third is ready for unit test.

Synchronizer

Frequency Generator

These units are complete except for the Bulova oscillator-discriminator unit. Unit test on the units has been accomplished, but, unit test on the assembly awaits a satisfactory output from the Bulova oscillator.

System #2 (less oscillator-discriminator) was delivered to the Model Shop on December 11, 1961 for assembly on the frame.

Synchronizer Generator

Additional rework is being done on the flight test unit to provide increased delay for the initiation of the gated 120 MC pulse to the buffer. This was necessary in order to decrease the buffer pulse width.

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System #2 was delivered to the Model Shop on December 11, 1961 for assembly on the frame.

Microwave Oscillator and Buffer Amplifier

One unit is now being used in composite test and except for self lock-up of the loop is operating satisfactorily.

The microwave oscillator, crystal modulator and klystron amplifier of the second unit have satisfactorily passed unit test.

Unit test has been completed on the phase detector amplifier of the third unit. Waveguide assembly is held up due to the lack of cross guide couplers.

Recorder

General

Recorder #1 was completely assembled and preliminary tests were conducted. A malfunction occurred which stopped the tests. Efforts to correct the malfunction have been underway and it is planned to conduct further tests during the second week of December.

Fabrication for units two and three continues.

Preliminary Tests

Preliminary test on the flight model recorder was made and completed satisfactorily. The final testing was scheduled for November 20, 1961. This test will be conducted using a mechanical sample of the fiber optics. The unit will, upon a satisfactory test, be delivered with the sample unit. When an acceptable unit becomes available the recorder will be returned for interchange of fiber optics.

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On November 20 the final test was started. The check of the D.C. voltages indicated trouble with the high voltage power supply. A check of the supply proved it was faulty and the final test was suspended.

Electrical

The high voltage power supply was returned to the vendor when it proved faulty in the tests. The trouble was caused by thermal run away of the switching transistors, evidently caused by the voltage on the 28 volt supply being too low. To avoid the possibility of this type of failure occurring again, the 1.0 ohm isolation resistor in the 28 volt ground return has been changed to 0.1 ohms.

Return of the corrected power supply is expected during the second week in December.

A separate interlock switch has been added to prevent the torque motors from spinning freely during CRT trace adjustment and film loading. This switch interrupts the 28 volts when the top cover is removed.

Fiber Optics

A close liaison has been maintained with Mosaic Fabrications, Inc., manufacturer of the fiber optic unfolding array, to determine the progress in solving the problems of nonuniformity in the geometry of the image. The most recent report indicates that the technical problem of making rectangular fiber bundles has been solved. It is expected that a sample bundle of rectangular fibers will be available for evaluation early in December.

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Discussions have also been held with other manufacturers of fiber optics during the report period. [REDACTED]

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Cathode Ray Tube Photography

At the present time, some tests were made of the overall resolution of the WX1745 CRT which has a nominal 1.0 mil spot, utilizing the recording equipment developed for the program. It is difficult to continue testing on this basis since the use of the recorder for this part of the program is capable of interfering with other facets of the program, such as evaluation of its performance when integrated with the remainder of the overall system. In order to avoid interferences of this kind, construction of a test set for photographic testing has begun. This set will be capable of providing results that can be directly transferred to the hardware program; e.g., selection of film emulsions based on sensitometric and resolution tests and evaluation of individual cathode ray tubes.

The test set will consist of the following basic components:

1. Deflection Circuitry, which will duplicate the circuitry in the hardware. Provision is also being made for a slow scan variable from below .1 inch per second to 5.0 inches per second, applied at right angle to the normal 5 KC trace so as to simulate various film speeds.
2. Locked Oscillator test component will supply a range of variable frequencies which can be locked to the synchronizing circuits of the deflection circuits. This will produce a stationary set of sine waves along the width sweep. The resulting recorded signal on film can then be read easily with a microdensitometer to determine degree of modulation.

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3. High Voltage Supply to operate the CRT must be capable of the same degree of regulation as the equipment used in the final package.
4. Miscellaneous Power Supplies to operate the above equipment will be supplied by using the present test set, constructed for the testing of the finished recorder. Since it is planned to move the recorder to a different locality of overall systems tests, this unit will be available.
5. It is certain that other components will be needed as CRT photographic testing proceeds. The test set will be flexible enough to accommodate such additions and changes that are deemed necessary.
6. The cathode ray tube mount will be designed so that photography of its display can be easily accomplished. As part of this subassembly, magnetic shielding is required.

Initially most of the work will be concerned with establishing the film requirements for a conventional lens recorder. Tests made to date indicate that the selection of a film emulsion with prerequisite characteristics of speed and resolution will be difficult. With the test set, films can be evaluated both for resolution and sensitometric characteristics with a minimum of interference to/or from the hardware development program.

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Navigation Tie-in

The first unit has been unit tested, the second unit is now in unit test and the third unit is being modified.

All of the servo and accelerometer amplifiers have been unit tested.

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The three accelerometers accepted from (Ref. previous report) were checked for frequency and linearity characteristics and found to be satisfactory. However, the gain of these units was 15 volts/g instead of 20 volts/g which necessitated modification of the accelerometer amplifiers to increase the gain. This modification work has been completed.

Power Supply and Control

Control Panel #1 and Power Supply #1 are available for composite test.

Power Supply #2 has been checked out in the altitude chamber to verify performance under the altitude and temperature requirements of the system. This unit has also been run through some exploratory vibration tests in all three axis. There appear to be no problems due to altitude, temperature or vibration.

Frame (Electrical)

Model Shop wiring of the second and third units is now in progress.

Frame (Mechanical)

Frame #2 has been delivered to the composite test area.

Frame #3 is 90% complete and will be completed on schedule for assembly of system #3.

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Truss

Preliminary design is complete and drafting will be started during the next reporting period.

Stress Analysis

The loads at installation mounting points are now available and will be submitted for customer consideration during the next reporting period.

System Interconnections

Cables for system #2 are complete. Cables for system #3 are 50% complete and will be completed on schedule for assembly in system #3.

Unit Test Cables

All unit test and patch cables have been completed.

System Handling Equipment

The second system test cart has been delivered to the composite test area.

Completion of the third cart is expected to be on schedule for composite test of system #3.

Composite Test

The modulator, duplexer driver, duplexer and variable frequency generator were added to system #1 composite test.

Preliminary tests indicated that the stalo locks up automatically.

Introduction of a simulated signal into the TWT produced what appeared to be a good double sided video signal from the video amplifier.

It is expected that installation of the sideband filter in the system will reduce the TWT noise figure from 12 down to 9 db.

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A reasonably good output was obtained from the modulator and KPA using the RF input from the receiver.

The synchronizer has been modified for the flight test program.

Efforts are now directed toward cleaning up signal leakage from the transmitter and synchronizer into the receiver.

System #2 is now being assembled.

Test Equipment

Composite Test Equipment

Work on the Composite Test Equipment has consisted of assistance in operation and setting up of the special test equipment.

Design Evaluation Equipment

The design, breadboarding, test and fabrication is progressing within the predicted schedule. Some problem areas exist which will be pointed out in the following paragraphs. At the present time it appears that the majority of the test equipment will be ready for delivery to the project by February 1, 1962.

Transponder

Although all of the items for the transponder have been ordered only a few items have been received. The lead time on most of these items is at least one month. The fabrication and test of the phase shifter and the 120 mc amplifiers for driving the crystal modulator has been completed. They will be ready for assembly when the rest of the ordered parts have been received.

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Clutter Generator

The noise source, which will produce a broad band of white noise as well as a narrow band of low frequency random noise that is flat to approximately 15 cps, has been tested. With the test equipment available it appears to be functioning properly. When the subsonic analyzer, becomes available, a plot of the output will be run to determine if it has the desired characteristics - a band of random noise flat from 0 to 15 cps.

The Gate Control Circuit and the Gate Circuit have been breadboarded and tested with the noise generator and are working satisfactorily.

Range Resolution Test Pattern Generator

The trigger amplifier has been fabricated and tested in its final package. The keyed oscillator and pulse generator have been fabricated and tested. Operation is completely satisfactory up to a pulse repetition rate of 50 mc. From 50 to 90 mc the pulse shape remains stable but the amplitude of the pulse degrades. It is felt that the output may be satisfactory when this unit is connected to the crystal switch. Since these tests cannot be run until a crystal switch is available, effort will continue to be expended to improve the output amplitude.

It should be noted that this unit is producing a 10 nsec pulse at a 90 mc repetition rate. This leaves approximately 1.1 Nsec interpulse recovery time for the generator. This is a severe requirement.

This unit is approximately 60% complete.

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Azimuth Resolution Test Pattern Generator

The $\div 128$ and $\times 128$ have been changed to $\div 256$ and $\times 256$.

These units have been fabricated in their final package. The $\times 256$ has been tested and is working satisfactorily. The test of the $\div 256$ was held up due to a delay in delivery of the necessary transistors and some diodes. These parts were received on the 12th and tests are proceeding.

The mixers have been designed and breadboarded and are working satisfactorily as modules. They have not as yet been tested with the complete breadboard chassis.

The design and fabrication of the SSB filters is being done by the Components group. The filters will be available for installation in the chassis on January 2, 1962.

The Tachometer, Tachometer amplifier, pulse position modulator and motor drive amplifier have been breadboarded and are working satisfactorily, open loop. The loop cannot be closed due to undelivered parts, such as gears, and the DC drive motor, from suppliers. Delivery of these parts has been re-scheduled for the last week in December 1961.

The final package of the tachometer was delivered by the Model Shop on the 14th.

The Ramp Generator and the Ramp Control have been breadboarded and are working satisfactorily. The linearity of the ramp over the necessary range of slopes is linear to better than $\pm 1/2\%$.

A servo set control has been added. This circuitry will hold off the sync pulse to the Range Resolution Test Pattern Generator, and therefore the modulation to the transponder, until the servo is ready to operate properly.

This chassis is approximately 40% complete.

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Azimuth Resolution Optics Assembly

The optical design of this unit has been firmed up and the initial optical layout is complete. The mechanical design of this unit is approximately 20% complete. All of the optical components in this category have been shipped on 12/6 by the suppliers.

This unit is approximately 30% complete.

Range Resolution and Dynamic Range Optics Assembly

The lower optics which contain the light source, microscope and movable table have been completely laid out and detailed by drafting. The drawings have been released to the model shop for fabrication. Some of the piece parts have been delivered by the model shop.

Approximately 70% of the upper optics and amplifier have been detailed and released to the model shop.

This unit is approximately 40% complete.

Film Evaluator Electronic Circuitry

The block diagram of the electronic evaluation circuits has been firmed up. The lamp power supply, the photo multiplier power supply and the photo multiplier circuit have been breadboarded and checked out. The counter circuits for the Range Resolution evaluation are being breadboarded at present in the lab.

The Evaluation Electronics are approximately 25% complete.

Mechanical Design and Packaging

Design of the overall layout of the mechanical package has begun. This has progressed to a preliminary sketch of the front panel arrangement, compilation of power supply loading and a preliminary compilation of the overall wiring.

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Navigation Data Recorder

During this reporting period, manufacture of required components was completed.

Trial assembly and wiring of the first unit was begun November 27, 1961. On December 6, the data chamber assembly was first operated and all step motors functioned properly.

Trial operation of the entire unit was started on December 11.

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SECRETAPPENDIX

Spatial Frequency Response of Cathode Ray Tubes

The purpose of this paper is to clarify the understanding of what happens when a sinusoidal intensity variation is reproduced by a Gaussian spot.

The effect is exactly the same as if the sine wave is scanned through a Gaussian aperture. The effect is represented mathematically by a convolution of the two functions.

Let the spot and the variation to be represented by

$$s(x) = e^{-\frac{x^2}{\sqrt{2}\sigma^2}} \quad (1)$$

and

$$v(x) = \cos \omega x \quad (2)$$

The convolution function may then be written as

$$F(x) = \int_{-\infty}^{\infty} s(y)v(x-y) dy \quad \text{or} \quad (3a)$$

$$F(x) = \int_{-\infty}^{\infty} v(y)s(x-y) dy \quad (3b)$$

Using (3a), we get

$$F(x) = \int_{-\infty}^{\infty} e^{-\frac{y^2}{\sqrt{2}\sigma^2}} \cos \omega(x-y) dy \quad (4)$$

$$= \cos \omega x \int_{-\infty}^{\infty} e^{-\frac{y^2}{\sqrt{2}\sigma^2}} \cos \omega y dy \quad (5)$$

$$= \cos \omega x \left[\frac{1}{\sqrt{2}} \sqrt{\pi} \sigma e^{-\frac{\omega^2 \sigma^2}{2\sqrt{2}}} \right] \quad (6)$$

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The factor in brackets represents the effect of the Gaussian on the sinusoidal pattern, i.e., it is the transfer function of the Gaussian

When $\frac{\omega^2 \sigma^2}{2} = 1$ this term will drop to 50% of its value at $\omega = 0$.

This occurs when

$$\omega = \frac{\sqrt{2}}{\sigma} \quad (7)$$

or the spatial frequency f (cycles/in) is

$$f = \frac{1}{\sqrt{2} \pi \sigma} = \frac{.45}{2\sigma} \quad (8)$$

where 2σ is the half-amplitude spot width.

An often used criterion is the limiting resolution. If this is taken arbitrarily as 10 db below peak, we

get
$$f_{10} = \frac{\sqrt{1.15} \sqrt{2}}{\pi \sigma} = \frac{.81}{2\sigma} \quad (9)$$

Note that this result is different than the impulse response. The case of two successive impulses gives a distribution which may be considered as the scanning of a Gaussian by another Gaussian, or, in terms of the convolution integral,

$$G(x) = \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}} e^{-\frac{(y-x)^2}{2\sigma^2}} dy \quad (10)$$

This reduces to

$$G(x) = \frac{\sigma \sqrt{\pi}}{\sqrt{2}} e^{-\frac{x^2}{2\sqrt{2}\sigma^2}} \quad (11)$$

The response is then
$$F(x) = \frac{\sigma \pi}{\sqrt{2}} \left(1 - e^{-\frac{x^2}{2\sqrt{2}\sigma^2}} \right) \quad (12)$$

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This has a value of 0.5 when $\chi = \sqrt{2} \theta$ corresponding to the peak-to-valley spacing of the resultant. Therefore the corresponding pulse spacing is $2\sqrt{2} \theta$ or in terms of pulse repetition frequency

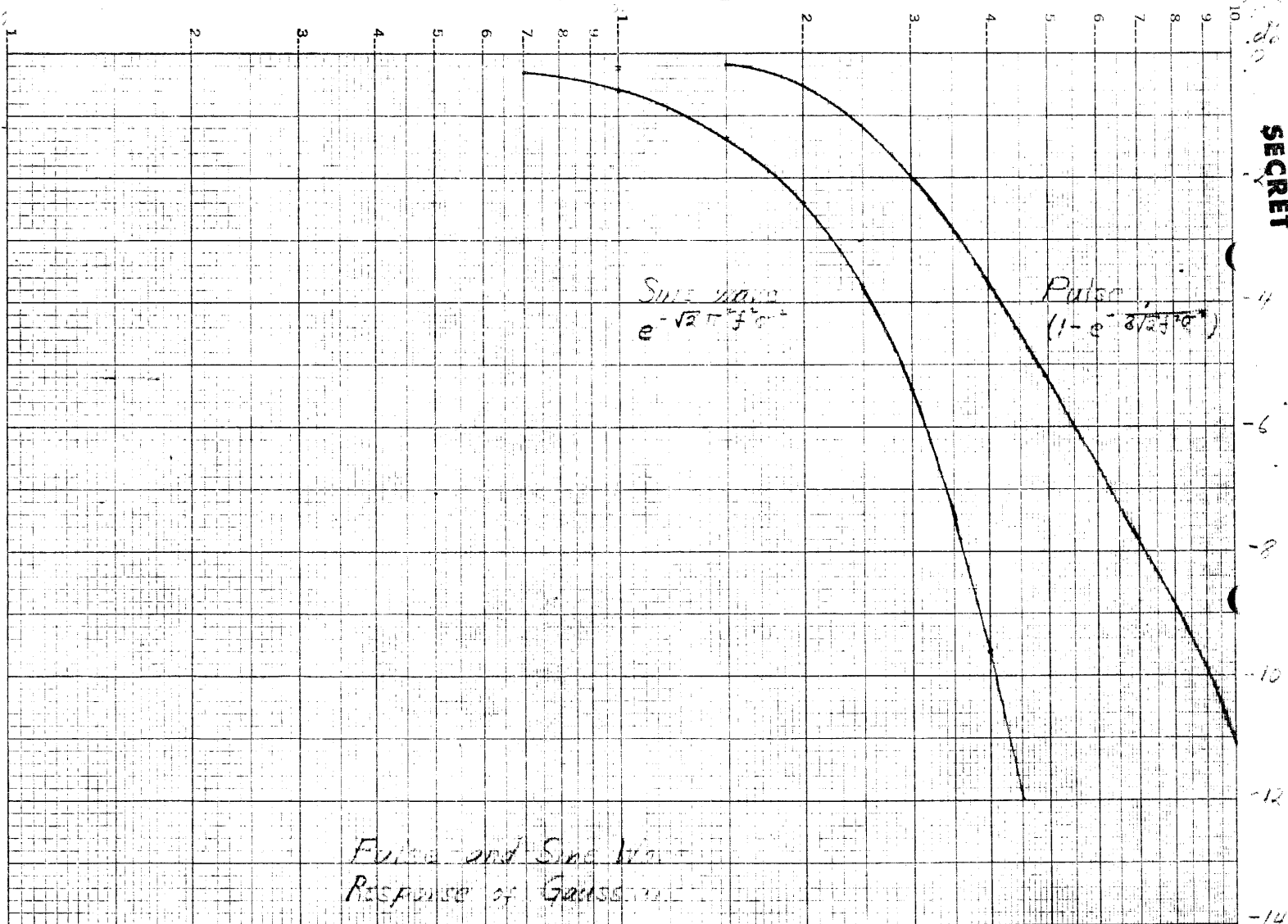
$$f_r = \frac{1}{2\sqrt{2} \theta} \quad (13)$$

The 10 db frequency can be found where $G(\chi) = 0.9G(0)$, or $\frac{\chi^2}{2\sqrt{2} \theta^2} = .1044$
 or $\chi = .542 \theta$. Therefore the 10 db frequency is

$$f_{10} = \frac{1}{1.084 \theta} = \frac{1.84}{2 \theta} \quad (14)$$

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AIR ARM DIVISION

BALTIMORE, MD., U.S.A.



Encl #2
OXC-3003
COPY 3 OF 3

TECHNICAL MEMOTITLE

FILTERING OF THE ERROR SIGNAL FOR A DOPPLER TRACKER

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UNTERM HEADINGS	PREPARED BY	DATE
		12/28/61
	PRI	
	CHECKED BY	25X1
	APR	12/28/61
	INCLUDE ILLUSTRATIONS	PAGE 1. OF 14 PAGES

SECURITY UNCLASSIFIED

This memo discusses some of the considerations which restrict the filtering of the angle error signal of a zero doppler tracker for a synthetic aperture radar. Such a tracker is used to correct for aircraft drift angle. Since the signal being tracked is typically a band of noise, the error signal is quite noisy, and must be heavily filtered to obtain the DC error component.

There are two effects which determine how complete this filtering must be, depending on the type of tracker used. If the angle error signal is used to aim the physical antenna, only the first effect is of interest. However, if the reference oscillator for the received signal is controlled by the error signal to give compensation, both effects are important. In both cases, the angle tracking (or frequency tracking) accuracy is obviously directly dependent on the filtering. (This is discussed in TM-38). In the second case, the noise on the error signal will modulate the reference oscillator. This is a much more serious effect than the first, since the reference deviations will tend to decorrelate the signal, so that focusing of the signal in the processor cannot take place properly. It is this effect which will be investigated here.

I. RATE LIMIT ON OFFSET FREQUENCY

In some case, it may be desirable to compensate for yaw or drift angles in a synthetic aperture radar by electronically shifting the offset frequency, rather than by antenna mechanical motion. There is a difference, however, in the two approaches, since mechanical antenna motion affects only the signal envelope without disturbing the detailed doppler waveform. The electronic compensation, on the other hand, affects the detailed waveform, and not the envelope. The result of the latter is much more serious, and must be limited to a tolerable amount. The rate at which the offset frequency can be changed without adverse effects will be discussed in this section.

The doppler frequency of a point target as a function of time is shown in figure 1, where the offset frequency is a constant of f_0 . If the offset is varied at a constant rate, the result is as in figure 2, where δ is the change in the

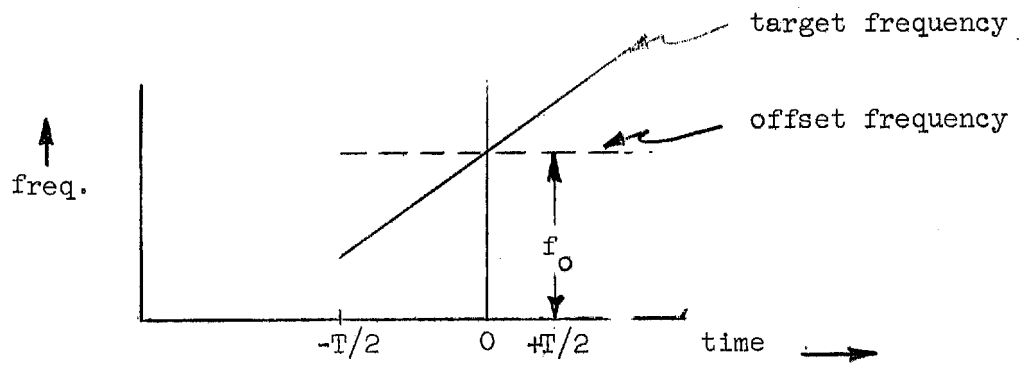


FIGURE 1

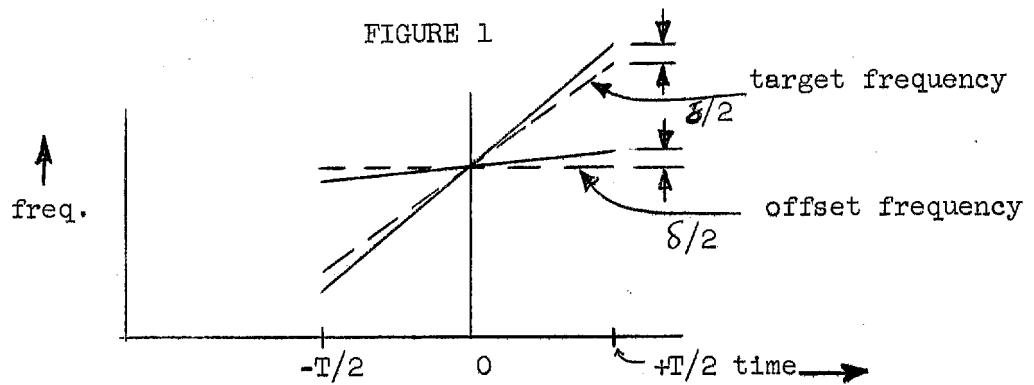


FIGURE 2

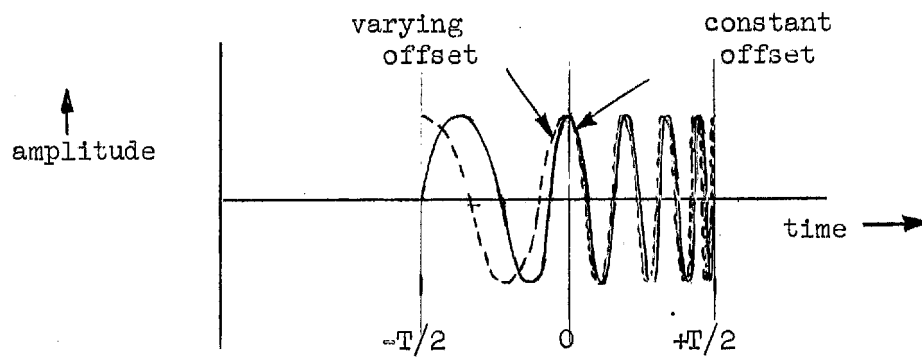


FIGURE 3

offset frequency in the dwell time T . The doppler waveform for the two cases is illustrated in figure 3. For the constant offset case, the doppler waveform is:

$$\text{Waveform}_1 = \cos \left[\frac{2\pi v^2 t^2}{R\lambda} + 2\pi f_o t \right] \quad -\frac{T}{2} \leq t \leq \frac{T}{2}$$

For the varying offset case, it is:

$$\text{Waveform}_2 = \cos \left[\frac{2\pi v^2 t^2}{R\lambda} + 2\pi \left(f_o + \frac{\delta}{T} t \right) t \right] \quad -\frac{T}{2} \leq t \leq \frac{T}{2}$$

The waveforms have been written in this form to make them in phase at time $t = 0$, so that the phase difference at the ends of the synthetic antenna ($t = \pm T/2$) can be found:

$$\begin{aligned} \text{Phase difference (from } t = 0 \text{ to } t = \frac{T}{2}) &= \left[\frac{2\pi v^2 (T/2)^2}{R\lambda} + 2\pi \left(f_o + \frac{\delta}{T} \frac{T}{2} \right) \frac{T}{2} \right. \\ &\quad \left. - \frac{2\pi v^2 (T/2)^2}{R\lambda} - 2\pi f_o \left(\frac{T}{2} \right) \right] \\ &= \frac{\pi \delta T}{2} \end{aligned}$$

The amount of phase difference which is tolerable must be set, so that δ can be found. From conventional antenna theory, quadratic phase error (from the center to either end of the antenna) results in a loss of main beam gain, beam broadening, and raising of the sidelobes. For a phase error of $\pi/2$ at the antenna ends the loss in main beam gain is about 1 db¹, which seems to be a reasonable value.

For random phase errors which are gaussianly distributed, a somewhat similar result occurs, with an RMS phase error of about 1/2 radian resulting in a main beam gain loss of 1 db².

For the synthetic antenna case, the operation is the same as for the conventional antenna, where the "antenna" length is now the aircraft velocity times the antenna dwell time. The requirements for the quadratic and random phase errors are then the same as given above for the real antenna.

¹ "Microwave Antenna Theory and Design" Rad Lab Vol 12, S. Silver.

² "Physical Limitations on Antenna", John Ruze, MIT Research Lab of Electronics Report #248

Limiting the phase error ($\pi\delta T/2$) to $\pi/2$, since this is a quadratic phase error, the tolerable rate of offset frequency change (δ/T) can be found:

$$\boxed{\delta/T = \frac{1}{T^2} \text{ (cps/sec)}} \quad \text{for a 1 db gain loss}$$

This can be converted to an equivalent angular rate, since:

$$f_D = \frac{2v}{\lambda} \theta$$

$$\frac{d\theta}{dt} = \frac{\lambda}{2v} \times \frac{\delta}{T} = \frac{\lambda}{2vT^2} \quad \text{(radians/sec)}$$

Another more intuitive, but perhaps more satisfying derivation of the allowable rate of offset is obtained by considering the processor as a bandpass filter of bandwidth $1/T$. If the VFO were to vary more than the bandwidth of the filter during the build-up time of the filter, the signal would be appreciably attenuated. Since the build-up time is T , the frequency rate allowable is $\frac{1}{T} \times \frac{1}{T} = \frac{1}{T^2}$, as before.

II. FILTERING REQUIRED TO LIMIT RATE OF OFFSET FREQUENCY

The simplified block diagram of a reference oscillator being controlled by the filtered doppler signal is shown in figure 4. The signal band of "noise", along with thermal noise is fed to a discriminator which converts the spectrum to a DC component (which is the error signal) plus a band of low frequency noise. A low-pass filter passes the DC unattenuated and rejects the higher frequency components of the noise. This voltage then controls the frequency of a variable-frequency oscillator such that the VFO instantaneous frequency displacement from its center frequency (f_0) is proportional to the voltage. This frequency is then used as a reference for the doppler signal for recording and processing. It is desired to find the bandwidth (b) of the low pass filter which will prevent the noise component of the voltage applied to the VFO from causing decorrelation of

the signal waveform.

In the previous section, the maximum rate (or slope) at which the VFO frequency may vary was established. A way must be found of finding the rate of VFO frequency change due to a known spectrum of noise applied to the VFO control element. The noise is assumed to have a gaussian distribution, which means that the slope can have any value from zero to plus or minus infinity. A reasonable approach is to find the RMS slope, and assume this is a typical value. In general, the larger slopes last for shorter times, and the smaller slopes for longer time. For the case where the bandwidth of the noise is much narrower than the reciprocal of the dwell time, the slope should be relatively constant over the dwell time. The slope requirement of $\frac{1}{T^2}$ found previously corresponded to a constant over the dwell time. No doubt greater slopes can be tolerated for lesser duration, so that the assumption of allowing the RMS slope to be $1/T^2$ seems reasonable, for the narrow-band noise condition.

The next step is then to determine the RMS slope of the frequency of the VFO. In Appendix I, the power spectrum of the slope of the VFO voltage is shown to be ω^2 times the power spectrum of the voltage. For a rectangular low pass spectrum of bandwidth b and total power σ_1^2 , the power spectrum of the slope is $P(f) = (2\pi f)^2 \frac{\sigma_1^2}{b}$ for $0 \leq f \leq b$. Taking the total power of this spectrum gives the mean square slope of $\frac{(2\pi b \sigma_1)^2}{\sqrt{3}}$. The resulting RMS slope of the VFO control voltage is $\frac{2\pi b \sigma_1}{\sqrt{3}}$ volts/second.

In terms of the parameters used in TM-38,

$$\sigma_1 = \sqrt{\frac{b}{\beta}} (S + N)$$

where

β = Bandwidth of doppler spectrum

S/N = Signal/noise ratio at input to discriminator. The transfer

characteristic of the VFO is B/S (cps/volt), therefore the RMS rate of change of frequency is B/S times the RMS slope of the VFO control voltage:

$$\text{RMS rate of change of freq} = \dot{f}_{\text{RMS}} = \left(\frac{2\pi b}{\sqrt{3}} \right) \left(\frac{B}{S} \right) \left(\sqrt{\frac{b}{S}} \right) (S+N)$$

$$\dot{f}_{\text{RMS}} = \frac{2\pi}{\sqrt{3}} \cdot b^{3/2} \cdot \sqrt{B} \cdot \left(1 + \frac{N}{S} \right)$$

To prevent decorrelation (assuming that the frequencies involved are low compared to $\frac{1}{T}$, $\dot{f}_{\text{RMS}} \leq \frac{1}{T^2}$, from the previous discussion.

Therefore the maximum bandwidth of the low pass filter is

$$b = \left[\frac{\sqrt{3}}{2\pi T^2 \sqrt{B} \left(1 + \frac{N}{S} \right)} \right]^{2/3} \quad \text{for 1 db loss, and } b \ll \frac{1}{T}$$

As an example, assume:

$T = 2$ seconds

$B = 150$ cps

$S/N = 0$ db

Solving for b gives $\frac{1}{50}$ cps. This restriction is obviously much more severe than was the angle (or frequency) tracking accuracy found in TM-38.

Furthermore, it is seen that the assumption was very good that the frequency of the noise components applied to the VFO are very low compared to the reciprocal dwell time.

III. FILTERING REQUIRED TO PREVENT EXCESSIVE PHASE DEVIATION

25X1

This section discusses an alternate way of looking at the filtering problem, suggested by Rather than determining the RMS rate-of-change of frequency due to noise, and referencing this to the maximum linear frequency rate, the allowable RMS phase deviation will be found. Although the procedure is quite different, and is based less on intuition, the final results will be found to be quite similar. The mathematical details will be found in Appendix II, but

the idea of the procedure will be given here. The problem with dealing with the phase function is that the actual phase of the modulation is not important, but only the variations of the phase during the antenna dwell time. Low-pass noise applied to an oscillator will give extremely large phase shifts, but these do not vary appreciably during a dwell time. The approach used here to eliminate this problem is to deal with a phase function (ψ) which is the difference between the actual phase and the average phase during a dwell time, i.e.:

$$\psi = \phi - \bar{\phi}_T$$

ϕ = actual phase

$\bar{\phi}_T$ = average of phase during a dwell time T

The problem is then to find the RMS value of ψ over a dwell time, for a given power spectrum of noise applied to the VFO control element. In the appendix, the transfer function $\frac{\psi}{E}$ of the VFO is found, where E is the input voltage to the VFO. The power spectrum of ψ is then the power spectrum of E times the magnitude of the transfer function squared. Integrating this power spectrum from 0 to b frequency gives the mean square value of ψ . The RMS of ψ is the square root of this value and is found to be:

$$\psi_{\text{RMS}} = \frac{\pi^2}{6\sqrt{3}} T^2 \left(1 + \frac{N}{S}\right) \sqrt{\beta} (b)^{3/2} \quad (\text{for } \pi b T \leq 1).$$

For a 1 db loss in synthetic antenna gain, ψ_{RMS} must be less than 1/2 radian. For this condition:

$$b = \left[\frac{3\sqrt{3}}{\pi^2 T^2 \sqrt{\beta} (1+N.S.)} \right]^{2/3} \quad \text{for } \pi b T \leq 1, \text{ and 1 db loss}$$

This is seen to be virtually identical with the frequency slope method result.

IV. SERVO LOOP

The closed-loop frequency tracker should contain an integrator so as to

have frequency memory during target fades, as for example, when flying over lakes or the like. Also, this will permit tracking the signal with no steady-state error as would otherwise result. (A steady-state frequency error would result in map inaccuracy.)

The feedback loop is shown in figure 5. The input frequency, consisting of signal and noise, is subtracted from the VFO frequency f_o , and used as an error signal to control f_o . The open loop has gain G and a single integrator $1/s$, where G represents the gain of the discriminator, integrator, and VFO; and $1/s$ represents the smoothing filter.

$$f_o = \frac{G/s}{1 + G/s} f_{in} = \frac{1}{1 + (1/G)s} f_{in}$$

The loop is then equivalent to a low pass RC filter of time constant $\gamma = \frac{1}{G}$. The noise bandwidth (b) of such a filter, in cps, is $\frac{1}{4\gamma}$ or $\frac{G}{4}$. Therefore the open loop gain should be made four times the value of b found in the previous sections.

$$G = 4 \left[\frac{\sqrt{3}}{2\pi T^2 \sqrt{B} (1 + \frac{N}{S})} \right]^{2/3}$$

Up to now, the input center-frequency has been assumed fixed, with the only effects considered being those due to the noise-like character of the signal, and thermal noise. As a rough indication of what sort of input frequency changes can be tolerated without decorrelating effects, consider first a step, and then a ramp of input frequency.

For a step of C , $f_o = C (1 - e^{-t/\gamma})$, and the maximum slope is C/γ at time 0, so that the maximum step for no decorrelation is $C = \frac{1}{T^2} = \frac{1}{4bT^2}$. For the previous example of $T = 2$ seconds, and $b = 1/50$ cps, $C = 3$ cps. Of course, this is a very small step, but then steps in frequency are not realistic.

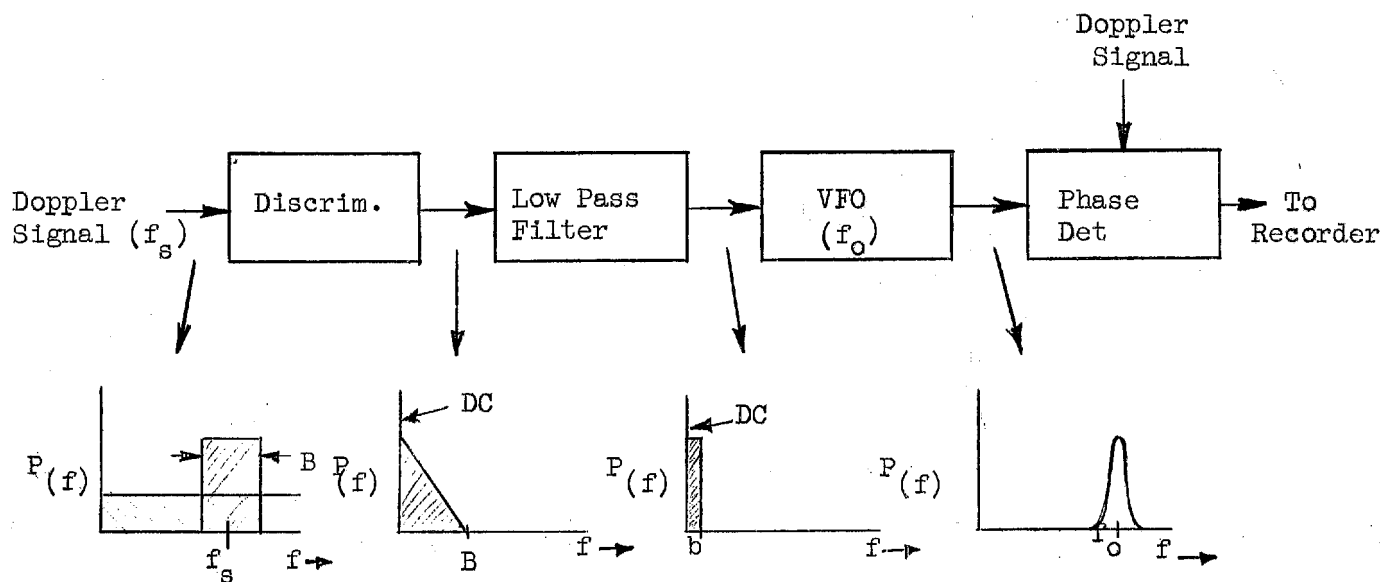


FIGURE 4

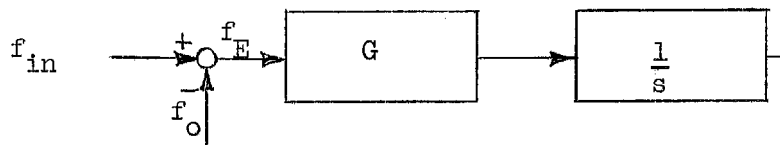


FIGURE 5

For a ramp of slope D , $f_o = D\tau \left[e^{-t/\tau} + t/\tau - 1 \right]$, so that the maximum slope is $D \left[1 - e^{-T/\tau} \right]$ at time T , so the maximum slope ramp allowable is $D = \frac{1}{T^2(1-e^{-4bT})}$. For the example, this gives a slope of 1.7 cps/second as the maximum input ramp which will not decorrelate the signal. Of course, these results are for an input which will not be corrected by other means.

APPENDIX ISlope of a Signal

It is desired to find the RMS slope of a noise-like signal. The power spectrum of the signal is known, and the power spectrum of the slope can be found as follows. The power spectrum (P'_f) of the output of a linear filter is

$$P'_f(f) = P(f) \times |T(f)|^2$$

where

$P(f)$ = power spectrum of input

$T(f)$ = voltage transfer function of the filter

Assume that the filter is an ideal differentiator (i.e. Laplace transform = s) then the differentiator output is the slope of the input.

$$P'_f(f) = P(f) \times |j\omega|^2$$

$$P'_f(f) = \omega^2 P(f)$$

To find the RMS slope, find the mean square slope and take its square root. The mean square slope is $\int_0^\infty P'_f(f) df$, so that the RMS slope is $\sqrt{\int_0^\infty P'_f(f) df}$. For the special case where the input spectrum is rectangular out to frequency b , of total power σ_1^2 , then:

$$P'_f(f) = (2\pi f)^2 \frac{\sigma_1^2}{b} \quad \text{for } 0 \leq f \leq b$$

$$\int_0^b P'_f(f) df = \frac{(2\pi)^2 \sigma_1^2 b^3}{3}$$

$$\sqrt{\int_0^b P'_f(f) df} = \frac{2\pi \sigma_1 b}{\sqrt{3}}$$

APPENDIX IIPHASE DEVIATION OF OSCILLATOR WITH NOISE ON CONTROL

$$\psi = \phi - \bar{\phi}_T$$

where:

ϕ = actual VFO output phase deviation

$\bar{\phi}_T$ = average phase deviation over dwell time T

$$\phi = \int \dot{\phi} dt$$

$\dot{\phi}$ = instantaneous angular frequency = $2\pi f$

$$f = \frac{B}{S} \times E$$

$\frac{B}{S}$ = scale factor of VFO in cps/volt

E = input voltage

$$\psi = \phi - \bar{\phi}_T = \int \dot{\phi} dt - \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} \left[\int \dot{\phi} dt \right] dt$$

$$= \frac{\dot{\phi}}{s} - \frac{\dot{\phi}}{s} \frac{1}{T} \left[\frac{e^{sT/2}}{s} - \frac{e^{-sT/2}}{s} \right]$$

$$= \frac{\dot{\phi}}{j\omega} \left[1 - \frac{\sin \pi f T}{\pi f T} \right]$$

$$T_{\psi}(f) = \frac{\psi}{E} = \frac{2\pi B}{j\omega S} \left[1 - \frac{\sin \pi f T}{\pi f T} \right] \text{ radians/volt}$$

where: $T_{\psi}(f)$ = transfer function of VFO

$$P_{\psi}(f) = P_E(f) |T_{\psi}|^2$$

where:

$P_{\psi}(f)$ = power spectrum of ψ

$P_E(f)$ = power spectrum of E

For $P_E(f)$ being a low pass rectangular spectrum of density $\frac{\sigma_1^2}{b}$ and bandwidth b,

$$P_{\psi}(f) = \frac{\sigma_1^2}{b} \frac{B^2}{S^2} \frac{1}{f^2} \left[1 - \frac{\sin \pi f T}{\pi f T} \right]^2 \text{ for } 0 \leq f \leq b$$

If the assumption is made that $\pi b T \leq 1$

$$P_{\psi}(f) \simeq \frac{\pi^4 \sigma_1^2 B^2 T^4}{36 b S^2} f^2$$

The mean square value of ψ is:

$$\psi_{MS} = \int_0^b P_{\psi}(f) df = \frac{\pi^4 \sigma_1^2 B^2 T^4 b^2}{108 S^2}$$

The RMS of ψ is the square root of ψ_{MS} :

$$\psi_{RMS} = \frac{\pi^2 \sigma_1 B T^2 b}{6\sqrt{3} S}$$

In TM-38 σ_1 was found to be $\sqrt{\frac{b}{B}} (S + N)$

$$\psi_{RMS} = \frac{\pi^2}{6\sqrt{3}} T^2 (1 + N/S) \sqrt{B} (b)^{3/2} \quad \text{for } \pi b T \leq 1$$

For an antenna, a gaussian distribution of random phase errors of 1/2 radian RMS error gives a loss in main beam gain of about 1 db. Since the synthetic antenna works the same as a real antenna, the 1/2 radian error is a reasonable design goal. For this performance

$$b = \left[\frac{3\sqrt{3}}{\pi^2 T^2 \sqrt{B} (1 + N/S)} \right]^{2/3} \quad \text{for } \pi b T \leq 1 \text{ and 1 db main beam loss}$$